$\boxed{\mathcal{AIPS}\text{ Memo 113}}$

Faceted imaging in $\mathcal{AIPS}$

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Abstract

"Image-plane faceting," in which each small image plane or facet is computed as tangent to the celestial sphere, has been the solution to the "W problem" in $\mathcal{AIPS}$ for some time. This memo describes another approach in which the facets are all in the same plane which is tangent to the sphere at the center of the field of view. This "uv-plane faceting" method may have some computational advantages and has replaced the `DO3DIMAG = FALSE` methods in $\mathcal{AIPS}$.

1 Basic concepts

The visibility *Vis* as a function of the baseline vector components is related with the source brightness distribution $B(l, m)$ (times the array element primary beam) by the expression (see Thompson et al., for example):

$$Vis(U_i, V_i, W_i) = \int B(l, m) \exp(j2\pi(U_i \cdot l + V_i \cdot m - \frac{1}{2}W_i \cdot (l^2 + m^2))dldm \tag{1}$$

where l, m are direction cosines of the vector to a point in the source picture plane;
U_i, V_i, W_i are the components of the baseline vector i;
B(l, m) is the source brightness distribution.

Having measured the set of the visibilities $(Vis(U_i, V_i, W_i), i = 1, 2...Nvis)$, we need to restore the two-dimensional source brightness distribution B(l, m). This task has a straight-forward solution if the term $\frac{1}{2}W_i \cdot (l^2 + m^2)$ is negligible. This requirement puts a limit on the maximum allowed l and m, limiting the field of view. To get around this limitation, one may divide the desired large field of view into a number of small images ("facets") each of which is small enough to allow the W term to be neglected.

$\mathcal{AIPS}$ has implemented a multi-faceted (`DO3DIMAG = TRUE`) scheme in which, for each facet, the U_i, V_i, W_i are rotated to the values they would have had if the observations were made with the facet center as the tangent point. The phases of the observed Vis are also rotated to the center of the facet. Both of these are implemented by straightforward 3x3 matrix multiplies during the gridding. Field rotation is also implemented through these matrices. Component subtraction requires similar multiplies to produce appropriate baseline components and adjusted phases. The separate small images, each tangent to the sphere at their center, along with their Clean components, remain the primary "image" and source model. However, for display and analysis, the separate facet images may be interpolated and averaged onto another larger grid, usually the tangent plane at the original phase stopping point. This is done with $\mathcal{AIPS}$ task `FLATN`.

If one could image instead each facet on a co-planar geometry, then the operation of `FLATN` could be simplified. It cannot be eliminated, since it is very unwise to use the image pixels along the edges and in the corners of each facet. Aliasing from the Fourier transform operation renders the edges unreliable and cumulative arithmetic errors are multiplied by very large correction functions in the corners rendering them even less reliable. $\mathcal{AIPS}$ allows users to place facets wherever they desire. However, the task `SETFC` will recommend placing the facets in a circular pattern on the sky with considerable overlap so the Clean need not extend outside an inscribed circle within each facet. `FLATN` will therefore be required to deal with the non-rectangular pattern of facet centers and with the overlaps even if the geometry is able to be co-planar.

We propose below a new (to $\mathcal{AIPS}$) way to arrange the mathematics that allows the facet images to be co-planar. It has been implemented as the `DO3DIMAG = FALSE` method in $\mathcal{AIPS}$, eliminating the previous, openly incorrect, method by that name. The only imaging task able to handle this new method is `IMAGR`; old tasks `MX` and `HORUS` were removed from $\mathcal{AIPS}$.

2 The new facet algorithm

We derive equation 1 using vector terminology. The visibility for the baseline vector $\vec{D}$ due to the brightness in the direction of the unit vector $\vec{e}$ is:

$$Vis(\vec{D}) = \int B(\vec{e}) \exp\ j2\pi(\vec{D} \cdot (\vec{e} - \vec{e_0}))\ d\vec{e} \tag{2}$$

Relation 2 is correct in any coordinate system so long as all vectors are in the same coordinate system. Let us chose the Cartesian coordinate system in which $\vec{u}, \vec{v}$ are in the tangent plane perpendicular to the vector $\vec{e_0}$ and vector $\vec{w}$ is along vector $\vec{e_0}$. Then

$$\begin{aligned}
\vec{D} &= \{U, V, W\} \\
\vec{e} - \vec{e_0} &= \{l, m, n\} \\
l &= \sin(\theta)\cos(\phi) \\
m &= \sin(\theta)\sin(\phi) \\
n &= 1 - \cos(\theta) \\
&= 1 - \sqrt{1 - \sin^2(\theta)} \\
&\sim \frac{1}{2}(l^2 + m^2)
\end{aligned}$$

Substituting the last equalities into equation 2, we arrive at equation 1 easily. Now remove the phase shift corresponding to the facet center by multiplying all visibilities by the relevant complex exponent:

$$\begin{aligned}
&Vis(U, V, W) \cdot \exp -j2\pi(U \cdot l_{i0} + V \cdot m_{i0} - \frac{1}{2}W \cdot (l_{i0}^2 + m_{i0}^2)) = \\
&\int B(l, m) \exp(j2\pi(U \cdot (l - l_{i0}) + V \cdot (m - m_{i0}) - \frac{1}{2}W \cdot ((l^2 - l_{i0}^2) + (m^2 - m_{i0}^2))dldm
\end{aligned} \tag{3}$$

where l_{i0}, m_{i0} are direction cosines of the vector directed to the center of the facet "i."

Introducing relative coordinates within the facet, $\Delta l_i = l - l_{i0}$ and $\Delta m_i = m - m_{i0}$, we obtain

$$\begin{aligned}
l^2 - l_{i0}^2 &= l_{i0}^2 + 2l_{i0}\Delta l_i + \Delta l_i^2 - l_{i0}^2 \\
m^2 - m_{i0}^2 &= m_{i0}^2 + 2m_{i0}\Delta m_i + \Delta m_i^2 - m_{i0}^2
\end{aligned} \tag{4}$$

The facet algorithm rule allows us to ignore both Δl_i^2 and Δm_i^2, simplifying equation 4 to:

$$\begin{aligned}
l^2 - l_{i0}^2 &= 2l_{i0}\Delta l_i \\
m^2 - m_{i0}^2 &= 2m_{i0}\Delta m_i
\end{aligned} \tag{5}$$

Using the equations 5 and the introduced relations $\Delta l_i = l - l_{i0} \quad \Delta m_i = m - m_{i0}$, we can convert equation 3 to the final relation between the brightness distribution in facet "i" and the measured visibilities:

$$Vis(U,V,W) \cdot \exp(-j2\pi(U \cdot l_{i0} + V \cdot m_{i0} - \frac{1}{2}W \cdot (l_{i0}^2 + m_{i0}^2)) =$$
$$\int B(\Delta l_i, \Delta m_i) \exp(j2\pi(U' \cdot \Delta l_i + V' \cdot \Delta m_i)\, dl_i dm_i \tag{6}$$

where

$$U' = U - W \cdot l_{i0}$$
$$V' = V - W \cdot m_{i0}$$

3 Improving the precision of the method

In the previous section, a simplified representation of the W term ($\frac{1}{2}W \cdot (l^2 + m^2)$) was used. To extend the analysis to a larger field of view, we present the analysis using the full correct representation of the W term: $-W(1 - \sqrt{1-(l^2+m^2)})$ Equation 3 should be rewritten:

$$Vis(U,V,W) \cdot \exp -j2\pi(U \cdot l_{i0} + V \cdot m_{i0} - W \cdot (1 - \sqrt{1-(l_{i0}^2 + m_{i0}^2)}) =$$
$$\int \exp(j2\pi(U \cdot (l - l_{i0}) + V \cdot (m - m_{i0}) + W \cdot \left(\sqrt{1-(l^2+m^2)} - \sqrt{1-(l_{i0}^2+m_{i0}^2)}\right)$$
$$B(l,m)\, dldm \tag{7}$$

where l_{i0}, m_{i0} are the direction cosines of the vector directed to the center of the facet "i"

We re-introduce $\Delta l_i = l - l_{i0}$ and $\Delta m_i = m - m_{i0}$ as positions relative to the center of facet "i.". Representing the difference of the square roots in equation 7 as a Taylor series, we include only the first order terms in Δl_i and Δm_i and omit the higher orders terms because of the facet algorithm rule. Thus

$$\begin{aligned}\left(\sqrt{1-(l^2+m^2)} - \sqrt{1-(l_{i0}^2+m_{i0}^2)}\right) &= \frac{\partial\sqrt{()}}{\partial l}\Delta l_i + \frac{\partial\sqrt{()}}{\partial m}\Delta m_i \\ &= -\frac{1}{\sqrt{1-(l_{i0}^2+m_{i0}^2)}}(l_{i0} \cdot \Delta l_i + m_{i0} \cdot \Delta m_i)\end{aligned} \tag{8}$$

Substituting equation 8 into equation 7, we can convert the latter to the final relation between the brightness distribution in facet "i" and the measured visibilities:

$$Vis(U,V,W) \cdot \exp -j2\pi(U \cdot l_{i0} + V \cdot m_{i0} - W \cdot (1 - \sqrt{1-(l_{i0}^2+m_{i0}^2)}) =$$
$$\int B(\Delta l_i, \Delta m_i) \exp(j2\pi(U' \cdot \Delta l_i + V' \cdot \Delta m_i)\, dl_i dm_i \tag{9}$$

where

$$U' = U - W \frac{l_{i0}}{\sqrt{1-(l_{i0}^2+m_{i0}^2)}};$$
$$V' = V - W \frac{m_{i0}}{\sqrt{1-(l_{i0}^2+m_{i0}^2)}}$$

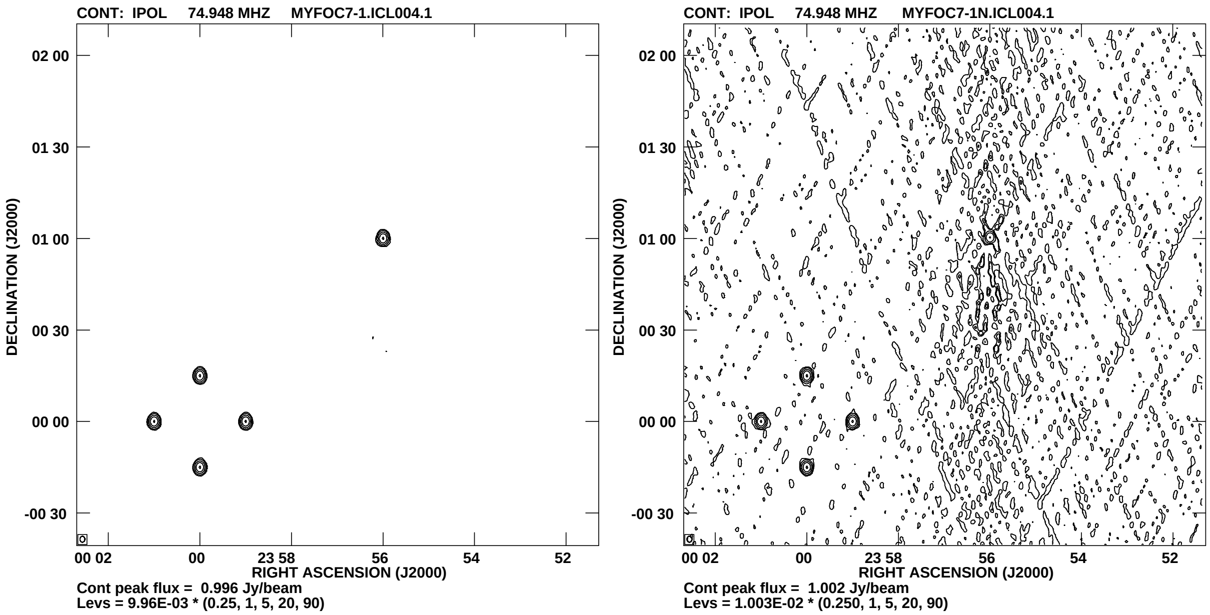

Figure 1: Model data imaged with `DO3DIMAG = FALSE` in `31DEC09` (left) and `31DEC08` (right) *AIPS*. The sources to the lower left were Cleaned in a central facet, only the source somewhat to the right and above the center was Cleaned in this facet. The image to the left is indistinguishable from one made with `DO3DIMAG = TRUE` except for the geometric differences.

4 Summary

The classic faceting algorithm computes a different coordinate system for each facet. In this coordinate system, the image plane is tangent to the celestial sphere at the facet center. As a result, the facet planes are not co-planar. The new faceting algorithm locates all facets on the same plane which is tangent to the celestial sphere at the center of field of view. In this case, all facets are co-planar. This should simplify the combination of the facets into a single large image for display and analysis, although issues of pixel overlap and reliability must still be handled.

In both faceting algorithms, the adjustment of the visibility phases and uv-plane coordinates are handled by 3x3 matrix multiplies. These allow for coordinate rotation about the facet center as well as correction for the W term. The matrix in the new algorithm has some terms which are zero, but not enough to justify using a more direct implementation. The fact that U and V now depend on facet, even when `DO3DIMAG = FALSE`, requires software which is more adaptable than the old routines used by `MX` and `HORUS`, requiring their elimination from *AIPS*. Because U and V depend on facet, the point-spread function ("dirty beam") of each facet is different from that of every other facet. These differences may have a small effect in the scaling of each facet and small effects in the finding of components within the inner cycle of Clean. The components are subtracted correctly from the residual visibility data, so the adaptive nature of Clean should minimize any errors in the minor cycles. Therefore, *AIPS* task `IMAGR` now allows the specification of `ONEBEAM` to allow Cleaning only with the dirty beam of the first facet. Tests suggest that the errors from this choice are not entirely corrected in later cycles leading to the suggestion that the slower `ONEBEAM FALSE; OVERLAP 2` methods be used while Cleaning the highest dynamic range portions of an image. Faster `ONEBEAM TRUE; OVERLAP 1` methods may be used in the later stages of Clean so long as no object is Cleaned in more than one facet.